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| RESEARCH ARTICLE

A Flexible Elasticity Framework for Measuring Firm Internationalization: Theoretical Model and Simulation-Based Evidence

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| ABSTRACT
This study develops a flexible elasticity framework for measuring firm internationalization. Unlike standard composite indices based on fixed weights, we model internationalization as a heterogeneous elasticity function in which the contribution of each dimension varies across firms. The model generalizes traditional additive approaches by introducing firm-specific elasticities that depend on observable characteristics of the firms. We provide a theoretical characterization of the model and evaluate its properties using simulations. The results show that fixed-weight indices systematically compress heterogeneity and distort firm rankings, whereas the proposed framework preserves structural variation and provides a more consistent representation of internationalization.

| KEYWORDS
Firm internationalization, heterogeneous firms, composite indices, measurement bias, flexible elasticity, simulation.

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1. Introduction

Firm internationalization is one of the central constructs in international economics and international business research. It captures the extent to which firms engage in cross-border activities and has been widely used to study globalization, productivity, export dynamics, and multinational production.

Despite its conceptual importance, measuring internationalization at the firm level remains methodologically challenging. The empirical literature typically relies on composite indices that aggregate multiple dimensions such as export intensity, foreign ownership, foreign sales, or employment abroad into a single scalar measure (Sullivan, 1994; Contractor et al., 2003). These indices are attractive because they reduce complex multidimensional behavior into a tractable empirical variable that can be easily incorporated into econometric models.

However, the construction of such indices relies on a critical and often implicit assumption: that the contribution of each dimension of internationalization is constant across firms. In other words, export intensity, foreign ownership, and firm size are assumed to enter the index with fixed weights that do not vary with firm characteristics. This assumption simplifies measurement but imposes a strong structural restriction on the data-generating process.

This restriction is increasingly difficult to reconcile with modern theories of heterogeneous firms and international trade. Since the seminal contribution of Melitz (2003), the literature has emphasized that firms differ systematically in productivity, size, and export behavior. Only a subset of firms self-select into international markets, and those that do tend to exhibit substantially

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different characteristics compared to non-internationalized firms. Subsequent work has extended this insight to multinational production and global sourcing decisions (Helpman et al., 2004; Antràs, 2003), showing that internationalization is not a binary or uniform process, but rather a heterogeneous and multi-channel phenomenon.

Empirical evidence further reinforces this view. Studies consistently find large differences between exporters and non-exporters in terms of size, productivity, capital intensity, and innovation capacity (Bernard et al., 2007). Moreover, firms engage in international markets through different mechanisms—direct exports, foreign direct investment, licensing, and participation in global value chains—each of which may have different economic determinants and implications.

Against this background, the use of fixed-weight composite indices becomes conceptually problematic. If firms differ not only in the level of internationalization but also in the way they internationalize, then imposing constant aggregation weights may distort the measurement of the underlying construct. In particular, fixed-weight indices may compress heterogeneity, reduce variability across firms, and generate systematic measurement bias correlated with firm size or productivity.

This paper addresses this issue by proposing a flexible elasticity framework for measuring firm internationalization. The key idea is to move away from fixed aggregation rules and instead model internationalization as a structural function in which the contribution of each dimension depends on firm-specific characteristics. In this framework, elasticities are not parameters fixed across all firms, but endogenous objects that vary with observable firm attributes such as size, sector, or ownership structure.

By doing so, the paper contributes to the growing literature that questions whether standard measurement practices in economics adequately reflect underlying heterogeneity. Rather than treating measurement as a neutral transformation of data, this paper views it as a structural modeling choice that can shape empirical conclusions.

The remainder of the paper develops this framework formally, evaluates its properties through simulation, and compares it with standard fixed-weight approaches. The results show that allowing for elasticity heterogeneity significantly changes both the distribution and ranking of firm-level internationalization, highlighting the importance of reconsidering how key economic constructs are measured in empirical research.

2. Literature Review

This section reviews the theoretical and empirical literature on firm internationalization, focusing on its measurement. It first discusses the conceptual foundations of internationalization as a firm-level construct and then examines the role of firm heterogeneity in international trade theory. It subsequently reviews the main empirical approaches used to measure internationalization and their limitations, with particular attention to aggregation issues. This section concludes by identifying the gap between theory and measurement that motivates the proposed framework.

2.1 Internationalization as a firm-level construct

The concept of firm internationalization has been widely studied across international business and international economics, yet its definition remains inherently multidimensional and conceptually diffuse. Early contributions framed internationalization as a gradual process of foreign market expansion driven by learning and uncertainty reduction. The Uppsala model (Johanson & Vahlne, 1977) conceptualizes internationalization as a sequential process in which firms progressively increase their commitment to foreign markets as experiential knowledge accumulates.

In parallel, the Eclectic Paradigm developed by Dunning (1980, 2001) provides a structural explanation of multinational activity based on ownership, location, and internalization advantages (the OLI framework). While highly influential, these frameworks primarily focus on explaining why firms internationalize rather than how internationalization should be measured empirically.

More recent contributions have challenged this gradualist perspective. The literature on international new ventures and "born global" firms highlights the existence of firms that internationalize rapidly from inception, often leveraging their technological capabilities and global networks. Network-based approaches further emphasize the role of inter-firm relationships in facilitating international expansion, suggesting that internationalization may be discontinuous, path-dependent, and shaped by external linkages.

As the empirical literature expanded, researchers increasingly required quantitative proxies to capture firm-level international engagement. This shift led to the development of composite indicators intended to summarize multidimensional behavior into a single scalar measure. However, this transition from conceptual theory to empirical measurement introduced a fundamental tension: while theory emphasizes heterogeneity and process, measurement tends to impose aggregation and uniformity.

2.2 Heterogeneous firms and the micro-foundations of international trade

A major transformation in international economics has been the incorporation of firm heterogeneity into trade theory. The seminal contribution of Melitz (2003) introduced a model in which firms differ in productivity, and only the most productive firms are able to bear the fixed costs associated with exporting. This framework generates endogenous selection into export markets and explains the highly skewed distribution of exporters observed in the data set.

Building on this foundation, Helpman, Melitz, and Yeaple (2004) extend the analysis to multinational production. Their model shows that firms choose between exporting and foreign direct investment depending on productivity levels and fixed costs, with more productive firms engaging in deeper forms of international integration than less productive firms.

Antràs (2003) further contributes by incorporating incomplete contracts into the theory of global production, emphasizing how contractual friction shapes the boundaries of the firm and the organization of cross-border activities.

Empirical evidence strongly supports these predictions. Bernard et al. (2007) document that exporting firms are systematically larger, more productive, more capital intensive, and more skill intensive than non-exporters. These differences are structural rather than marginal, indicating that internationalization is inherently firm heterogenous.

Despite this rich theoretical and empirical literature, most contributions focus on explaining participation in international markets (extensive margin) rather than measuring the intensity and structure of internationalization (intensive margin).

2.3 Measurement approaches in international business

Given the multidimensional nature of internationalization, empirical research has relied heavily on proxy variables and composite indices. One of the most widely used measures is export intensity, which is typically defined as the ratio of exports to total sales. Although simple and intuitive, this measure captures only one dimension of international engagement.

To address this limitation, Sullivan (1994) proposes a multidimensional index that combines export intensity, foreign sales, and geographic dispersion. Subsequent contributions, such as Contractor et al. (2003), develop composite indices that aggregate multiple dimensions of firm international activity, using predetermined weights.

Other commonly used indicators include foreign sales to total sales (FSTS), foreign assets to total assets, and broader indices such as the Transnationality Index developed by UNCTAD, which combines sales, assets, and employment across countries. These measures aim to capture different facets of internationalization, including its scale, scope, and geographic diversification.

Despite their widespread use, these approaches share a common limitation: they rely on fixed or exogenously determined weights across firms. This implies that each component of internationalization contributes equally—or according to an arbitrary rule—regardless of firm characteristics such as size, productivity, or industry. Consequently, these measures may provide only a partial and potentially misleading representation of firm-level international engagement.

2.4 Critiques of composite indices and aggregation bias

A growing strand of literature has begun to critically examine the limitations of composite indices in economics and management research. A central concern is aggregation bias, which arises when heterogeneous components are combined using fixed weights that do not reflect underlying structural differences.

In the context of internationalization, this issue is particularly severe because different dimensions of global engagement—exports, foreign ownership, foreign employment—are likely to have different marginal contributions depending on firm-specific characteristics. For example, export intensity may be highly informative for small firms but less so for large multinational enterprises, where foreign subsidiaries play a more important role.

When fixed weights are imposed, the resulting index may compress variation across firms and artificially homogenize internationalization patterns. This can lead to measurement error that is not random but systematically related to firm characteristics such as size or productivity.

From an econometric perspective, this type of measurement error is particularly problematic because it is likely to be non-classical. If the measurement bias is correlated with explanatory variables commonly used in empirical analysis, standard estimators may produce biased and inconsistent results.

Although the literature on index construction has highlighted these concerns in related contexts (e.g., productivity measurement, development indices, and innovation metrics), relatively little attention has been devoted to internationalization measures specifically.

2.5 Structural measurement and the role of heterogeneity

An emerging perspective in economics emphasizes that measurement should be consistent with underlying structural models. Rather than treating indices as arbitrary aggregations, this approach derives measurement systems from theoretical frameworks that explicitly incorporate heterogeneity across economic agents (Heckman, 2001; Browning, Hansen, and Heckman, 1999).

Applied to internationalization, this perspective highlights a key limitation of standard indices: the implicit assumption of a common aggregation structure across firms. If firms differ in how they combine export activity, foreign ownership, and scale into international engagement, then imposing fixed weights amounts to assuming a restrictive and potentially misspecified functional form.

This insight is closely related to the literature on heterogeneous firms (Melitz, 2003; Helpman et al., 2004). If firm characteristics, such as productivity and scale, influence both the mode and intensity of internationalization, then the measurement should allow for interaction effects between firm attributes and internationalization channels.

More generally, recent work in applied microeconometrics has stressed the importance of flexible functional forms and heterogeneous effects when modeling firm behavior (e.g., Athey and Imbens, 2017). This suggests the need for measurement frameworks in which the contribution of each component of internationalization can vary across firms rather than being imposed ex ante.

Despite this conceptual alignment, most empirical measures remain disconnected from structural trade models, leaving a persistent gap between theory and actual measurement.

2.6 Research gap and contribution

The literature reviewed above highlights three main research gaps.

First, existing internationalization theories provide rich explanations of firm behavior but do not offer explicit frameworks for measurement (Johanson and Vahlne, 1977; Dunning, 2001).Second, heterogeneous firm models emphasize cross-firm variation but do not translate this heterogeneity into empirically implementable measurement systems (Melitz, 2003; Helpman et al., 2004).Third, widely used empirical indices rely on fixed aggregation rules that are inconsistent with both theoretical predictions and observed firm heterogeneity (Contractor et al., 2003; Sullivan, 1994).

To the best of our knowledge, no existing measure of firm-level internationalization explicitly allows firm-specific elasticities in the aggregation of international activities. This represents a critical limitation, as it prevents empirical measures from aligning with the core insights of the heterogeneous firm models.

This study contributes to the literature by proposing a flexible elasticity framework that integrates heterogeneity directly into the measurement of internationalization, in line with recent calls for structurally grounded empirical approaches (Heckman, 2001). Instead of imposing fixed weights, the model allows the elasticities to vary across firms as a function of observable characteristics.

In doing so, this study bridges the gap between theoretical models of heterogeneous firms and empirical measurement practices in international business, providing a more consistent and flexible approach to capturing firm-level international engagement.

3. Theoretical Model

Building upon the conceptual framework outlined in the introduction, we formally define our flexible elasticity model for firm internationalization. The model departs from traditional additive approaches by conceptualizing internationalization as a multiplicative function of key dimensions, allowing for heterogeneous contributions across components. Our core innovation lies

in introducing firm-specific elasticities, which are not fixed but instead dynamically determined by observable firm characteristics. This section presents the mathematical structure of the model, starting from its general form and then detailing the endogenous nature of elasticities, culminating in a log-linear representation suitable for analysis.

We define firm internationalization as:

$$II_i = X_{1i}^{\theta_{1i}} \cdot X_{2i}^{\theta_{2i}} \cdot X_{3i}^{\theta_{3i}}$$

where:

- X_{1i}: export intensity
- X_{2i}: foreign ownership
- X_{3i}: firm size

The choice of export intensity, foreign ownership, and firm size as the three core dimensions of internationalization is motivated by their complementary role in capturing distinct mechanisms of global engagement. Export intensity reflects direct participation in international markets, foreign ownership captures integration into multinational production and governance structures, while firm size proxies the scale at which international activities can be sustained and coordinated. Importantly, the interaction among these dimensions is central to the model's mechanism, as internationalization emerges from their joint configuration rather than from any single component in isolation.

Firm size is included as a dimension of internationalization because it captures the scale of a firm's global operations. Larger firms typically operate across multiple markets, manage more complex international networks, and sustain higher levels of cross-border activity. In this sense, firm size reflects not only a structural characteristic but also a manifestation of international engagement.

At the same time, firm size enters the model through the elasticity structure, as described below, where it influences how firms combine different internationalization channels. This dual role reflects the fact that size operates both as a level effect (scale of international activity) and as a factor shaping the relative importance of each component.

3.1 Endogenous elasticities

A central tenet of our framework is the notion of endogenous elasticities, which allows the marginal contribution of each internationalization dimension to vary systematically across firms. Unlike fixed-weight models, we posit that the elasticities (α_i) are not constant parameters but rather functions of observable firm characteristics (Z_i), such as firm size, sector, or country of origin. This formulation directly addresses the heterogeneity problem by embedding firm-specific attributes into the aggregation process, reflecting the structural differences in how firms leverage various international activities. Elasticities vary across firms:

$$\theta_{ki} = \theta_k + Z_i' \phi_k$$

where:

- Z_i: firm characteristics (size, sector, country)
- ϕ_k: heterogeneity parameters

This linear specification is chosen for tractability and transparency. It provides a first-order approximation of more general heterogeneous elasticity structures.

3.2 Log-linear representation

$$\log(II_i) = \sum_k \theta_k \log(X_{ki}) + \sum_k (\log(X_{ki}) \cdot Z_i' \phi_k)$$

This representation highlights that internationalization depends not only on observable inputs but also on interactions between these inputs and firm characteristics. While the baseline specification assumes a linear relationship between $Z_i$and the elasticity parameters, alternative nonlinear functional forms (including simple quadratic specifications in Z_i) were also considered in preliminary robustness checks. These alternatives do not qualitatively affect the main results in terms of dispersion, ranking instability, or measurement bias. For this reason, the linear specification is retained as a parsimonious and interpretable first-order approximation of a potentially richer heterogeneity structure.

3.3 Interpretation

Marginal effects:

$$\frac{\partial \log\ II_i}{\partial \log\ X_{ki}} = \theta_k + Z_i'\phi_k$$

Thus, internationalization is not fixed but structurally heterogeneous.

4. Theoretical Properties

The flexible elasticity framework developed in this study departs from standard approaches to measuring firm internationalization by embedding heterogeneity directly into the aggregation structure of international activity. Rather than treating internationalization as a fixed-weight combination of observable components, the model allows the contribution of each dimension—such as exports, foreign ownership, and firm size—to vary systematically across firms.

This structural relaxation has important theoretical implications. First, it challenges the implicit assumption in most composite indices that the marginal contribution of each internationalization channel is firm-independent. Second, it introduces a mechanism through which firm characteristics shape not only the level of internationalization but also the way in which different international activities are combined into a single measure.

From a theoretical perspective, this implies that internationalization should not be understood as a scalar outcome generated by a uniform technology of aggregation, but as a heterogeneous transformation process in which firms differ in their "conversion rates" between observable international activities and overall global engagement. This interpretation is consistent with the heterogeneous firm literature, which emphasizes that firms differ not only in productivity levels but also in the structure of their engagement in international markets (Melitz, 2003; Helpman, Melitz, & Yeaple, 2004).

In this context, the standard Cobb-Douglas or linear aggregation approaches can be interpreted as special cases of a more general structural model in which elasticities are constrained to be identical across firms. Relaxing this restriction allows for a richer characterization of internationalization patterns and provides a bridge between theoretical models of firm heterogeneity and empirical measurement.

Therefore, the purpose of this section is to formally characterize the key theoretical properties of the proposed framework. In particular, we derive three main results that clarify the relationship between the flexible elasticity model and existing approaches: (i) the model nests standard Cobb-Douglas specifications as a limiting case; (ii) introducing elasticity heterogeneity increases the dispersion of internationalization measures; and (iii) heterogeneous elasticities generate nonlinear aggregation effects even when the underlying functional form is log-linear.

Together, these properties highlight the structural differences between fixed-weight indices and the proposed framework, and provide the theoretical foundation for the subsequent simulation analysis.

4.1 Proposition 1: Nesting property (Cobb-Douglas benchmark)

Proposition 1. The proposed model nests the Cobb-Douglas specification as a special case under homogeneous elasticities.

Formally, consider the following general specification:

$$II_i = X_{1i}^{\theta_{1i}} X_{2i}^{\theta_{2i}} X_{3i}^{\theta_{3i}}$$

where:

$$\theta_{ki} = \theta_k + Z_i'\phi_k$$

If heterogeneity is absent, that is,, the elasticities collapse to constants:

$$\theta_{ki} = \theta_k$$

Substituting into the model yields the following:

$$II_i = X_{1i}^{\theta_1} X_{2i}^{\theta_2} X_{3i}^{\theta_3}$$

This is the standard Cobb-Douglas functional form widely used in production theory and index aggregation (Dunning, 1980; Solow, 1957).

4.1.1 Economic interpretation

This nesting property is important for two reasons:

First, it shows that the proposed model does not discard standard approaches but generalizes them instead. The Cobb-Douglas index can be interpreted as a restricted version of the model, where firms are assumed to be homogeneous in how they combine internationalization channels.

Second, it clarifies that the fixed-weight indices used in the empirical literature implicitly impose a strong homogeneity restriction on firm behavior. In this sense, standard indices can be viewed as degenerate cases of richer structural models.

4.2 Proposition 2: Heterogeneity increases dispersion

Proposition 2. Introducing firm-level heterogeneity in elasticities increases the dispersion of internationalization measures relative to the fixed-weight indices.

To examine this, consider the variance of the log internationalization:

$$\log (II_i) = \sum_k \theta_{ki} \log (X_{ki})$$

Under fixed weights:

$$\theta_{ki} = \theta_k$$

so variation in II_iarises only from variation in X_{ki}.

Under heterogeneous elasticities, we have:

$$\theta_{ki} = \theta_k + Z_i' \phi_k$$

variation arises from both:

- variation in inputs X_{ki}
- variation in elasticities Z_i

This implies:

$$Var(\log II_i^{flex}) > Var(\log II_i^{fixed})$$

provided that $Var(Z_i' \phi_k) > 0$.

4.2.1 Economic interpretation

This result has clear economic implications. In traditional indices, all firms are assumed to aggregate internationalization channels similarly. As a result, differences across firms reflect only differences in the levels of exports, ownership, or size.

In contrast, the flexible elasticity model allows firms to differ not only in levels but also in the *marginal valuation* of each channel. This introduces an additional layer of heterogeneity that increases the dispersion.

This is consistent with empirical findings in the heterogeneous firm literature, where differences between firms are not only quantitative but also structural (Melitz, 2003; Bernard et al., 2007).

4.3 Proposition 3: Nonlinear aggregation effects

Proposition 3. Even though the model is log-linear in inputs, the presence of heterogeneous elasticities generates nonlinear aggregation effects at the level of internationalization measurement.

Consider the log transformation as follows:

$$\log (II_i) = \sum_k \theta_{ki} \log (X_{ki}$$

Substituting elasticities:

$$\log(II_i) = \sum_k \theta_k \log(X_{ki}) + \sum_k (Z_i' \phi_k) \log(X_{ki})$$

The second term introduces the interaction effects between firm characteristics and internationalization channels.

4.3.1 Key implication

Even though the functional form appears linear in logs, the presence of Z_i-dependent coefficients implies that the mapping from $X_{ki}$to $II_i$is **nonlinear in the space of firms**.

In particular:

- changes in X_{ki}have heterogeneous effects across firms
- marginal effects depend on the level of Z_i
- aggregation is no longer additive in a structural sense

4.3.2 Economic interpretation

This result is crucial for interpreting the internationalization measures. Standard indices assume that a proportional increase in exports or foreign ownership has the same effect on all the firms. The proposed framework demonstrates that this is generally false.

Instead, the impact of increasing exports depends on firm characteristics, such as size and ownership structure. Large firms may translate export growth into internationalization differently than small firms do.

This is consistent with the idea that internationalization is not a scalar transformation but a firm-specific process shaped by internal structure and external constraints.

4.4 Additional property: scale dependence (extended implication)

A further implication of the model is that internationalization becomes scale-dependent.

Since:

$$\theta_{ki} = \theta_k + Z_i' \phi_k$$

and Z_i often includes firm size. Thus, we have:

$$\frac{\partial \log II_i}{\partial \log X_{ki}} \neq constant$$

This implies that:

- large firms may exhibit stronger or weaker sensitivity to exports depending on φ
- the same change in exports produces different outcomes across firm sizes
- aggregation bias increases with firm heterogeneity in size distribution

This property connects the model directly to the heterogeneous firm literature and reinforces the idea that firm size is not only a level effect but also a structural moderator of the intensity of internationalization.

4.5 Summary of theoretical insights

The theoretical properties of the model can be summarized as follows:

1. **Nesting property**: standard Cobb-Douglas indices are special cases
2. **Heterogeneity amplification**: dispersion increases when elasticities vary
3. **Nonlinearity in aggregation**: interactions emerge even in log-linear form
4. **Scale dependence**: firm size affects marginal contributions of internationalization channels

5. Simulation Design

We simulate a cross-section of heterogeneous firms following standard distributional assumptions commonly used in the international trade and heterogeneous firms literature (Melitz, 2003; Helpman et al., 2004). The objective of the simulation design is not to replicate a specific empirical dataset, but to construct a controlled environment in which the implications of alternative measurement strategies can be clearly identified and compared.

The simulation framework is designed to capture three stylized facts documented in the empirical literature on firm internationalization. First, firm size and productivity are highly skewed, with a small number of large firms accounting for a disproportionate share of economic activity. Second, export participation and foreign ownership are heterogeneous across firms, with only a subset engaging intensively in international markets. Third, different dimensions of internationalization are correlated but not perfectly aligned, reflecting the multidimensional nature of global engagement.

To incorporate these features, we generate a population of firms with heterogeneous characteristics drawn from standard statistical distributions used in the literature. Firm size is modeled using a Pareto distribution, consistent with the empirically observed heavy-tailed distribution of firm size, where a small number of firms dominate aggregate economic activity.

Export intensity is generated using a log-normal distribution, capturing the positive skewness and multiplicative nature of export performance across firms. This specification reflects the idea that export outcomes arise from multiplicative shocks related to productivity, demand conditions, and market access.

Foreign ownership shares are modeled using a beta distribution, which naturally constrains values between zero and one while allowing for flexible shapes, including skewness and bimodality. This is particularly suitable for capturing heterogeneous patterns of multinational ownership across firms.

In addition, we generate a vector of firm-specific characteristics Z_i, which enter the elasticity structure of the model. These characteristics are drawn from a standard normal distribution and are interpreted as reduced-form proxies for unobserved structural determinants (e.g., managerial quality, firm-specific intangible assets, or institutional embeddedness) that modulate the effectiveness of internationalization channels. For the purpose of the simulation, Z_i represents a generic firm-specific attribute influencing elasticity variation.

The simulation design is conducted in two stages. In the first stage, firm-level observable variables are generated according to the distributions described above. In the second stage, internationalization outcomes are constructed using both a fixed-weight aggregation approach and the proposed flexible elasticity specification. This dual construction allows for a direct comparison between traditional measurement approaches and the heterogeneous elasticity framework.

The fixed-weight benchmark index is defined as a linear combination of export intensity, foreign ownership, and firm size with exogenously imposed weights. These weights are typically chosen based on normalization or empirical convenience, as is standard in the literature.

In contrast, the flexible elasticity index is constructed using a multiplicative structure in which elasticities vary across firms as a function of their observable characteristics. This introduces endogenous heterogeneity into the aggregation process and allows the contribution of each dimension of internationalization to systematically differ across firms.

While the baseline simulation relies on standard distributional assumptions, additional robustness checks were conducted by varying the shape and scale parameters of the underlying distributions (e.g., adjusting the Pareto exponent by ±20% and the Beta distribution's α, β parameters by ±10%). These unreported checks consistently yielded qualitatively similar results, confirming the robustness of our main findings regarding increased dispersion, ranking instability, and systematic bias across a range of plausible firm characteristic distributions.

By comparing these two measurement approaches within the same simulated environment, the analysis isolates the role of the aggregation structure in shaping observed patterns of internationalization. In particular, it allows us to distinguish between variation driven by underlying economic heterogeneity and variation induced by measurement design.

Finally, the simulation framework is sufficiently general to allow for extensions. In particular, it can accommodate alternative distributional assumptions, additional firm characteristics, and alternative functional forms for elasticity heterogeneity. This flexibility ensures that the results are not driven by restrictive assumptions and enhances the external validity of the simulation exercise.

5.1 Data generating process

- Export intensity: log-normal
- Foreign ownership: beta distribution
- Firm size: Pareto distribution
- Characteristics Z_i: normal distribution

5.2 Competing measures

Fixed-weight index

$$II_i^{fixed} = 0.5X_{1i} + 0.3X_{2i} + 0.2X_{3i}$$

Flexible elasticity index

$$II_i^{flex} = X_{1i}^{\theta_{1i}} X_{2i}^{\theta_{2i}} X_{3i}^{\theta_{3i}}$$

6. Nomenclature / List of Symbols

6.1 Variables

- II_i: Firm internationalization index
- X_{1i}: Export intensity of firm i
- X_{2i}: Foreign ownership share of firm i
- X_{3i}: Firm size
- Z_i: Vector of firm-specific characteristics (e.g. size, sector, country)

6.2 Parameters

- θ_{ki}: Firm-specific elasticity associated with dimension k
- θ_k: Baseline elasticity for dimension k
- ϕ_k: Heterogeneity parameter vector associated with dimension k

6.3 Model structure

- $k \in \{1,2,3\}$: Internationalization dimensions
- i: Firm index

6.4 Key equations

- $II_i = \prod_k X_{ki}^{\theta_{ki}}$: Flexible elasticity index
- $\theta_{ki} = \theta_k + Z_i'\phi_k$: Endogenous elasticity specification
- $\log(II_i) = \sum_k \theta_k \log(X_{ki}) + \sum_k \log(X_{ki}) Z_i'\phi_k$: Log-linear form

7. Simulation Results

This section presents the results of the simulation exercise comparing the flexible elasticity framework developed in Section 3 with a standard fixed-weight index. The objective is to evaluate how introducing endogenous, firm-specific elasticities affects the measurement of internationalization in terms of dispersion, ranking stability, and systematic measurement distortion.

All simulations are based on the data-generating process specified in Section 3, where firm internationalization is defined as:

$$II_i = \prod_k X_{ki}^{\theta_{ki}}, \theta_{ki} = \theta_k + Z_i'\phi_k$$

While the baseline specification relies on standard distributional assumptions commonly used in the literature, alternative parameterizations of the underlying distributions produce qualitatively similar results across all main outcomes (unreported robustness checks).

7.1 Dispersion

A first key result is that the flexible elasticity framework generates higher dispersion in internationalization measures relative to the fixed-weight index. This result is a direct consequence of heterogeneity in elasticities. Since:

$$\theta_{ki} = \theta_k + Z_i'\phi_k$$

elasticities vary systematically across firms, introducing an additional source of cross-sectional variation in the composite index. As a result, the variability of II_iincreases, implying:

$$Var(\theta_{ki}) > 0 \;\Rightarrow\; Var(\log\, II_i^{flex}) > Var(\log\, II_i^{fixed})$$

To quantify this effect, we compute the variance and interquartile range of both indices, as well as the dispersion ratio:

$$DR = \frac{Var(II_i^{flex})}{Var(II_i^{fixed})}$$

The results consistently indicate that $DR > 1$across all simulation runs, confirming that the flexible specification systematically amplifies cross-firm heterogeneity.

This amplification is particularly strong in the upper part of the distribution. Due to the multiplicative structure of the model, firms with higher elasticities experience stronger nonlinear effects, leading to greater dispersion at the top of the internationalization distribution.

Overall, these results indicate that fixed-weight indices mechanically compress variation by imposing homogeneous elasticities across firms, whereas the flexible framework restores heterogeneity that is structurally embedded in firm characteristics.

7.2 Ranking effects

We next examine how endogenous elasticities affect firm rankings.

To assess ranking stability, we compute Spearman's rank correlation coefficient between the two measures:

$$\rho = Corr(Rank(II_i^{flex}), Rank(II_i^{fixed}))$$

The results show that rank correlations are strictly below one, indicating systematic reordering of firms when moving from the fixed-weight to the flexible framework.

This ranking instability arises from the interaction between firm characteristics and internationalization components. Since:

$$\theta_{ki} = \theta_k + Z_i'\phi_k$$

the effective weight assigned to each X_{ki}depends on Z_i. Therefore, even if two firms have similar levels of X_{ki}, differences in $Z_i$alter their aggregate index.

As a result, ranking changes are driven not only by differences in observed inputs, but also by structural heterogeneity embedded in elasticities. This leads to non-trivial ranking reversals, particularly among mid-ranked firms, where small differences in $Z_i$generate meaningful changes in effective weights.

These findings imply that conventional fixed-weight indices are not invariant to aggregation structure and may distort cross-firm comparisons in heterogeneous samples.

7.3 Measurement bias

Finally, we analyze systematic differences between the fixed-weight index and the flexible elasticity measure.

We define firm-level measurement bias as:

$$Bias_i = II_i^{fixed} - II_i^{flex}$$

This bias captures the deviation introduced by imposing homogeneous elasticities in the fixed-weight specification, when in fact the true aggregation structure is heterogeneous and depends on firm characteristics.

Importantly, this bias does not arise from stochastic measurement error, but from **structural misspecification of the aggregation function**. In the flexible framework, elasticities vary systematically with firm characteristics:

$$\theta_{ki} = \theta_k + Z_i'\phi_k$$

whereas the fixed-weight index imposes $\theta_{ki} = \theta_k$for all firms. As a consequence, the fixed-weight measure fails to account for heterogeneity in the marginal contribution of each internationalization dimension.

The results show that this bias is not random but systematically related to firm characteristics. In particular, firms with higher values of $Z_i$tend to exhibit larger absolute deviations between the two measures, reflecting differences in the implied elasticities.

To characterize this pattern, we analyze the distribution of $Bias_i$and its relationship with observable firm attributes. The results indicate:

- non-zero average bias,
- substantial cross-firm variation,
- systematic correlation with firm characteristics such as size and international exposure.

This implies that the fixed-weight index introduces a **systematic measurement bias**, which may distort empirical analysis when such indices are used in regressions or benchmarking exercises. In particular, estimated relationships based on II_i^{fixed}may reflect aggregation misspecification rather than true economic effects.

By contrast, the flexible elasticity framework mitigates this distortion by allowing the aggregation weights to adjust endogenously to firm heterogeneity, thereby providing a more structurally consistent measure of internationalization.

8. Empirical Considerations

To complement the simulation evidence, this section provides an illustrative empirical discussion of how the flexible elasticity framework could be operationalized using firm-level data on international activity. The objective is not to estimate a structural model, but to assess whether the qualitative implications of the framework are consistent with patterns typically observed in empirical studies of firm internationalization.

We consider two alternative measures of firm internationalization: (i) a standard fixed-weight index commonly used in the literature, and (ii) a flexible elasticity index that allows weights to vary with observable firm characteristics.

The fixed-weight index is defined as a linear aggregation of export intensity, foreign ownership, and firm size, consistent with standard empirical practice. The flexible index, in contrast, assigns heterogeneous weights that depend on firm characteristics such as size and sectoral affiliation, in line with the theoretical framework developed in Section 3.

Due to data limitations, the flexible elasticity structure is not directly estimated. Instead, it is interpreted in reduced-form terms, where observable firm characteristics (such as size class and sector dummies) are used as proxies for the underlying heterogeneity in elasticities.

The comparison between the two measures highlights three qualitative patterns consistent with the theoretical predictions. First, the flexible specification implies greater dispersion in internationalization outcomes relative to the fixed-weight index, reflecting the role of heterogeneity in aggregation. Second, the two measures would not produce identical firm rankings, implying that the choice of weighting scheme is likely to affect benchmarking exercises. Third, differences between the two indices are expected to vary systematically across firms, particularly along dimensions associated with international exposure.

Overall, this section is intended as a conceptual empirical illustration rather than a structural estimation exercise. The observed patterns are consistent with the mechanism of heterogeneous elasticities proposed in the theoretical model, but do not constitute causal or structural empirical evidence.

9. Discussion

The results of the simulation exercise highlight several important implications for the measurement of firm internationalization. First, the flexible elasticity framework systematically increases the observed dispersion of internationalization measures relative to standard fixed-weight indices. This suggests that traditional aggregation methods may compress meaningful heterogeneity across firms by imposing homogeneous weights on structurally diverse units.

Second, the introduction of endogenous elasticities generates substantial instability in firm rankings. Since the effective contribution of each internationalization dimension depends on firm-specific characteristics, even small differences in these characteristics can lead to non-trivial reordering of firms. This has important implications for empirical studies that rely on ranking-based comparisons, benchmarking exercises, or top-firm classifications.

Third, the analysis reveals the presence of systematic measurement bias in conventional fixed-weight indices. This bias is not random but structurally related to firm characteristics such as size and international exposure. As a result, standard indices may systematically overstate or understate the degree of internationalization for specific groups of firms, introducing distortions in empirical analysis.

From an econometric perspective, these findings point to a form of non-classical measurement error, in which measurement distortions are systematically correlated with firm characteristics. Consequently, the error structure induced by fixed-weight aggregation is likely to be correlated with explanatory variables commonly used in empirical models. While the flexible elasticity framework reduces structural misspecification in the construction of the index, it does not eliminate potential endogeneity concerns arising from the determinants of firm heterogeneity. In particular, if the variables driving heterogeneous elasticities are jointly determined with firm outcomes, standard empirical strategies may still suffer from biased inference.

An additional econometric challenge arises from the potential endogeneity of the firm-specific characteristics $Z_i$entering the elasticity function. In empirical applications, variables such as firm size, R&D intensity, or foreign ownership may themselves be jointly determined with internationalization outcomes. This implies that $Z_i$may be correlated with unobserved determinants of II_i, leading to biased estimates of the heterogeneity parameters $\phi_k$if estimated using standard regression techniques.

To address this issue, a possible empirical strategy is a two-step procedure. In the first step, baseline elasticities $\theta_k$can be estimated using a standard fixed-weight specification or a reduced-form approximation of the internationalization index. In the second step, the residual variation in the constructed index can be regressed on $Z_i$to recover estimates of ϕ_k. Alternatively, an instrumental variables approach can be used, relying on lagged firm characteristics, industry-level averages, or policy-induced variation as instruments for Z_i. For instance, while lagged firm size or sectoral averages of R&D intensity are potential instruments, their validity would require careful justification and testing in specific empirical contexts. Future work needs to critically evaluate the practical feasibility and robustness of these strategies.

These considerations highlight the importance of appropriate econometric strategies when using composite internationalization measures in empirical work. Depending on the research design, instrumental variables approaches, panel data methods, or fixed-effects specifications may be required to address potential endogeneity and obtain consistent estimates.

The log-linear specification adopted in the model can be interpreted as a first-order approximation to a more general nonlinear structure. In this sense, it corresponds to a local linearization of the relationship between firm characteristics and internationalization outcomes. While richer nonlinear interactions may arise in practice—for example due to complementarities between export intensity, foreign ownership, and firm size, or the presence of threshold effects—the linear formulation provides a tractable baseline that captures the main sources of heterogeneity in a transparent and economically meaningful way.

Overall, the flexible elasticity framework provides a more structurally consistent approach to measuring firm internationalization by explicitly incorporating heterogeneity in aggregation weights. However, its use in empirical applications requires careful attention to identification issues and the potential endogeneity of the underlying determinants of heterogeneity.

10. Policy Implications

The results of this study have important implications for policy design, firm-level benchmarking, and the interpretation of internationalization indicators used in both academic and institutional contexts. Composite indices of internationalization are widely used by international organizations, development agencies, and policymakers to assess firm performance and globalization exposure (World Bank, 2020; OECD, 2013). Thus, the findings directly affect how these indicators should be constructed and interpreted.

The first implication concerns the design of firm-level benchmarking tools. Institutions such as the World Bank, OECD, and UNCTAD frequently rely on aggregate measures of export intensity or multinational activity to compare firms across countries and sectors (UNCTAD, 2013; World Bank, 2020). The results of this study suggest that such comparisons may be systematically biased if fixed-weight aggregation schemes are used, consistent with broader concerns about measurement distortions in composite indices (Sullivan, 1994; Contractor et al., 2003). In particular, firms with different structural characteristics may be incorrectly ranked, leading to distorted benchmarking outcomes and misleading cross-country comparisons.

The second implication relates to industrial and trade policy. Governments often use internationalization indicators to identify firms that are globally competitive or to design export promotion programs (Baldwin 2016; Cattaneo et al. 2013). However, the evidence in this study suggests that fixed-weight measures compress heterogeneity across firms, potentially misrepresenting the true structure of international engagement. This is particularly relevant in the context of global value chains, where firms participate through multiple channels beyond direct exports (Gereffi et al., 2005; Antràs and Chor, 2013). As a result, policies targeting based solely on export intensity may underestimate the role of firms that are highly internationalized through foreign ownership or production fragmentation.

The third implication concerns development policy and structural transformation strategies. In many developing and emerging economies, export-led growth strategies rely heavily on the assumption that export intensity is a sufficient proxy for international integration. However, the literature on global value chains shows that international integration is multidimensional and includes production networks, input-output linkages, and offshoring activities (Baldwin & Lopez-Gonzalez, 2015; World Bank, 2020). The flexible elasticity framework proposed in this study suggests that focusing exclusively on exports may lead to incomplete or biased assessments of firm-level globalization, particularly in economies where participation in global value chains is more important than direct exporting.

The fourth implication concerns policy evaluation and impact assessment. Empirical policy studies often use composite internationalization indices as outcome variables to evaluate trade liberalization, export promotion policies, and industrial interventions (De Loecker & Goldberg, 2014; Aghion & Howitt, 2009). If these indices are affected by structural measurement bias, then the estimated policy effects may reflect changes in measurement sensitivity rather than true behavioral responses. This introduces a form of non-classical measurement error that may systematically bias policy evaluation results.

Finally, the results suggest implications for statistical agencies and international organizations responsible for economic measurements. Reliance on fixed-weight aggregation rules may be too restrictive in contexts characterized by strong firm heterogeneity. More flexible approaches, potentially grounded in structural modeling or data-driven weighting schemes, could improve the accuracy of internationalization indicators and enhance their policy relevance (OECD, 2013; World Bank, 2020).

Overall, the policy implications extend beyond measurement issues and affect how governments and international organizations interpret firm-level globalization, design interventions and evaluate economic performance.

11. Conclusion

This study proposes a flexible elasticity framework for measuring firm internationalization. The main objective is to move beyond traditional composite indices based on fixed weights, which implicitly assume homogeneous contributions of export intensity, foreign ownership, and firm size across firms.

The model generalizes standard measurement approaches by allowing the elasticities to vary across firms as a function of observable characteristics. This formulation introduces a structural interpretation of internationalization as a heterogeneous process rather than a fixed linear aggregation of components.

The simulation results provide consistent evidence that fixed-weight indices systematically compress cross-firm heterogeneity. In contrast, the flexible elasticity model preserves the dispersion in internationalization measures and generates meaningful

differences in firm rankings. These differences are not random but are structurally linked to firm characteristics, such as size and exposure to international markets.

From a methodological perspective, this study's key contribution is to show that measurement can be a source of systematic bias. When aggregation rules impose constant weights, a part of the observed distribution of internationalization is mechanically constrained. This has important implications for empirical research in international economics and international business, where composite indices are widely used as explanatory and dependent variables.

The framework also suggests that internationalization should not be interpreted as a single scalar construct, but rather as a multidimensional process with heterogeneous marginal contributions. This is consistent with modern theories of heterogeneous firms and global production networks, where firms differ not only in whether they internationalize but also in how they do so.

Although this study is theoretical and simulation-based, the framework is directly applicable to microdata sources such as the World Bank Enterprise Surveys or firm-level trade datasets. Future research should empirically estimate the proposed model and test whether elasticity heterogeneity is systematically related to firm size, productivity, or sectoral characteristics.

In addition, further extensions could explore dynamic versions of the model, allowing the elasticities to evolve over time as firms accumulate international experience. Another promising direction is the integration of machine learning techniques to estimate heterogeneous elasticities in high-dimensional data settings.

Overall, the flexible elasticity framework contributes to a more general and structurally consistent method of measuring firm internationalization. By relaxing the assumption of fixed aggregation weights, the model opens new possibilities for both theoretical and empirical research in the field of international economics.

Data and Code Availability: The data used in this study are simulated and generated within the framework of the model described in Section 3. As such, no external dataset is employed. The code used to generate the simulations and replicate the results is available from the author upon reasonable request.

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Limitations

This paper has several limitations that should be acknowledged.

First, the simulation-based nature of the analysis implies that the results are not derived from a calibrated structural model or from direct estimation on firm-level microdata. While the simulation design is grounded in standard distributional assumptions from the heterogeneous firms literature, it is intended as a proof-of-concept illustrating the implications of alternative aggregation structures rather than as an empirically estimated model. Future research could pursue calibration using detailed firm-level microdata, for instance, from national statistical agencies or comprehensive international business surveys, to quantify the real-world impact of this framework more precisely.

Second, the specification of the elasticity functions relies on a linear functional form in observable firm characteristics. Although this assumption provides transparency and tractability, it may be restrictive. Future work should explore more flexible functional forms, including interaction effects or threshold models, as these could capture critical non-linearities in how elasticities vary with firm size or international exposure, potentially yielding different quantitative implications. For instance, firms might exhibit threshold effects where elasticities change drastically beyond certain size or international exposure levels. While our qualitative findings regarding increased dispersion and ranking instability are likely robust, future research should explore alternative formulations to assess the quantitative robustness of these results and capture richer real-world dynamics.

Third, while additional (unreported) robustness checks using alternative distributional parameterizations confirm the qualitative stability of the main findings, the simulation framework does not exhaust all possible empirical configurations. In particular, different joint distributions of firm characteristics may affect the magnitude, although not the direction, of the results.

Fourth, the framework abstracts from dynamic considerations and focuses on a static cross-sectional setting. In reality, firm internationalization is a dynamic process, and elasticities may evolve over time. Extending the model to a panel setting would be a useful direction for future research.

Finally, although the framework highlights potential issues related to measurement bias and endogeneity, it does not provide a full structural solution to causal identification problems. As discussed in Section 9, empirical applications of the proposed measure require appropriate econometric strategies such as instrumental variables or panel data methods—to address potential endogeneity in firm characteristics and ensure valid causal inference.

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