Journal of Mathematics and Statistics Studies
ISSN: 2709-4200
DOI: 10.32996/jmss
Journal Homepage: www.al-kindipublisher.com/index.php/jmss

JMSS
AL-KINDI CENTER FOR RESEARCH AND DEVELOPMENT

| RESEARCH ARTICLE

A Bazilevic function related to the class of meromorphic functions

Mays S. Abdul Ameer[1], Abdul Rahman S. Juma[2] and Hassan Hussien Ebrahim[3]

[1,3]*Department of Mathematics, College of Education for Pure Sciences, Tikrit University, Salahaddin, Iraq.*
[2]*Department of Mathematics, College of Education for Pure Science, University of Anbar, Ramadi, Iraq.*
Corresponding Author: **Mays S. Abdul Ameer**, **E-mail**: mays33@st.tu.edu.iq

| ABSTRACT

The goal of this article is to further geometric function theory by focusing on meromorphic functions and relying on the Bazilevic function, which represents the interactions of analytic functions. It focuses on presenting and investigating a differential operator for meromorphic functions. Using this operator, create meromorphic Bazilevic functions, a new subclass in the punctured unit disk $\psi = \{\zeta \in \mathbb{C}: 0 < |\zeta| < 1\}$. This text also greatly expands knowledge and understanding of the meromorphic function to fall into this new subclass.

| KEYWORDS

Bazilevic functions, Meromorphic functions, Differential operator.

| ARTICLE INFORMATION

ACCEPTED: 01 August 2025 **PUBLISHED:** 17 September 2025 **DOI:** 10.32996/jmss.2025.6.4.2

1. Introduction

Geometric function theory is a subfield of complex analysis that studies the geometric characteristics of analytic functions. The foundation of complex analysis is academic research on univalent and multivalent function theory. find it fascinating because of its complex geometry and variety of research options. Understanding univalent functions is crucial for the complicated analysis of single and multiple variables. Koebe's 1907 essays served as the inspiration for this concept [12]. Bieberbach in 1916 and Gronwall in 1914. Additional advancements in the field, including Bieberbach's second coefficient estimate, Gronwall's Area theorem, and Koebe's seminal book [5], were made possible by their work. By that time, univalent function theory was an established subject of research. Miller, Sanford S., and Petru T. Mocanu first proposed the idea of differential subordination. The notion of differential subordination was first presented by Mocanu and Miller in their 2000 book [3]. In 2003, they proposed differential superordination as a supplementary technique [15]. Several articles link meromorphic Bazilevic functions to a linear operator in complex analysis. This association enables the study of these functions using the operator's properties, resulting in new criteria and characterizations of the functions in the class. Mathematicians like Attiy and colleagues [3], Bulboaca and Zayed [14], Oros and Oros [18]. The Bazilevic meromorphic functions methodology has been used extensively by Abdulnabi et al. [1], Juma [10], and others.

Assume that the open unit disc is represented by $\psi\backslash\{0\} = \{\zeta \in \mathbb{C}: 0 < |\zeta| < 1\}$, and let T represent the class of meromorphic functions of the following form:

$$g(\zeta) = \frac{1}{\zeta} + \sum_{\kappa=2}^{\infty} a_\kappa \zeta^\kappa, \quad (\kappa = 1,2, \dots) \qquad (1)$$

which are normalized with $g(0) = 0$ and $g'(0) = 1$ and are analytic in the open unit disk.

If a function $g(\zeta)$ satisfies the following criteria, it is said to as meromorphic starlike of order η in ψ:

$$-\mathcal{R}e\left\{\frac{\zeta g'(\zeta)}{g(\zeta)}\right\} > \eta, (\zeta \in \psi, 0 \leq \eta < 1).$$

Additionally, if a function $g(\zeta)$ satisfies the following criteria, it is referred to as meromorphic close to convex order η in ψ.

$$-\mathcal{R}e\left\{\frac{\zeta(g(\acute{\zeta}))}{f(\zeta)}\right\} > \eta, (\zeta \in \psi, 0 \leq \eta < 1).$$

The differential operator defined on space T, is now introduced as follows (see [11]):

$$\mathcal{H}_{\xi,\mu}^{\nu,\rho}: \mathrm{T} \longrightarrow \mathrm{T}$$

$$\mathcal{H}_{\xi,\mu}^{\nu,\rho}\left(\left(g(\zeta)\right)^{\tau}\right) = (g(\zeta)^{\tau}).$$

$$\mathcal{H}_1 = \frac{(\rho-\xi)\zeta}{\mu+\rho}\mathcal{H} + \left(\frac{1-(\rho-\xi)}{\mu+\rho}\right)$$

$$\mathcal{H}_{\xi,\mu}^{1,\rho}\left(\left(g(\zeta)\right)^{\tau}\right) = \mathcal{H}_1(\mathcal{H}_{\xi,\mu}^{0,\rho}\left(\left(g(\zeta)\right)^{\tau}\right)) = \mathcal{H}_1[\zeta^{\tau} + \textstyle\sum_{\kappa=2}^{\infty} a_{\kappa}(\tau)\,\zeta^{\tau+\kappa-1}]$$

$$\mathcal{H}_{\xi,\mu}^{1,\rho}\left(\left(g(\zeta)\right)^{\tau}\right) = \left(\frac{1+(1-\tau)(\rho-\xi)}{\mu+\rho}\right)^{1}\zeta^{\tau} + \textstyle\sum_{\kappa=2}^{\infty}\left(\frac{1+(\kappa-\tau)(\rho-\xi)}{\mu+\rho}\right)^{1} a_{\kappa}(\tau)\,\zeta^{\tau+\kappa-1}$$

$$\mathcal{H}_{\xi,\mu}^{\nu,\rho}\left(\left(g(\zeta)\right)^{\tau}\right) = \mathcal{H}_1\left(\mathcal{H}_{\xi,\mu}^{1,\rho}\left(\left(g(\zeta)\right)^{\tau}\right)\right) = \left(\frac{1+(1-\tau)(\rho-\xi)}{\mu+\rho}\right)^{2}\zeta^{\tau} + \textstyle\sum_{\kappa=2}^{\infty}\left(\frac{1+(\rho-\xi)(\tau+\kappa-2)}{\mu+\rho}\right)^{2} a_{\kappa}(\tau)\,\zeta^{\tau+\kappa-1}.$$

In general, we have

$$\mathcal{H}_{\xi,\mu}^{\nu,\rho}\left(\left(g(\zeta)\right)^{\tau}\right) = \mathcal{H}_1(\mathcal{H}_{\xi,\mu}^{\nu-1,\rho}(\left(g(\zeta)\right)^{\tau}))$$

$$= \frac{1}{\zeta} + \left(\frac{1+(1-\tau)(\rho-\xi)}{\mu+\rho}\right)^{\nu}\zeta^{\tau} + \textstyle\sum_{\kappa=2}^{\infty}\left(\frac{1+(\rho-\xi)(\tau+\kappa-2)}{\mu+\rho}\right)^{\nu} a_{\kappa}(\tau)\,\zeta^{\tau+\kappa-1}. \qquad (2)$$

where $(\mu > 0, \xi, \rho \geq 0, \nu \in N_0 = N \cup \{0\};\ \tau > 0, \zeta \in \psi)$.

Complex analysis, Bazilevic functions are an important subclass of univalent analytic functions. I. E. Bazilevic, a Soviet mathematician, first proposed it in 1955 [13]. Exploring larger families of univalent functions that display intricate angular distortions and rotational behaviors was the original driving force behind Bazilevic functions. By combining geometric and analytical methods, the Bazilevic construction method opened up new avenues for the study of conformal and quasi-conformal mappings. Since their debut, mathematicians have become more interested in Bazilevic functions because of their rich structure and broad range of applications in geometric function theory. Researchers have expanded the theory over the years, investigated a number of subclasses, and looked at how they behaved under various operators and transformations. Bazilevic functions are still being studied today, and they have applications in computational mathematics and mathematical modeling in addition to pure mathematics fields like differential equations, geometric mapping theory, and functional analysis. Their enduring significance highlights their fundamental contribution to the evolution of contemporary complex analysis, see([8],[18],[19],and [20]). He presented and examined this function as follows:

$$g(z) = \left\{\frac{\xi}{1+\varepsilon^2}\int_0^z (h(v) - i\varepsilon)v^{-(1+\frac{i\xi\varepsilon}{1+\varepsilon^2})}\, g(v)^{\frac{\xi}{1+\varepsilon^2}}\, dv\right\}^{\frac{1+i\varepsilon}{\xi}}, \qquad (3)$$

where the function $h(v)$ belongs to T and $g(v)$belongs to starlike function, with $\beta > 0$.

Next, use the differential operator to define a class of Bazilevic functions (see [10]).

Definition 1.1. A function $g(\zeta)^{\tau}$ is called Bazilevic meromorphic and order η, if it satisfies the following:

$$-\mathcal{R}e\left\{\frac{\zeta\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho} g(\zeta)^{\tau}\right)'}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho} g(\zeta)^{\tau}\right)^{1-\Im}\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho} f(\zeta)^{\tau}\right)^{\Im}}\right\} > \eta, \qquad (4)$$

where $(\Im \geq 0, 0 \leq \eta < 1, \nu \in \mathbb{N}_0, \rho \geq 0, \zeta \in \psi)$.

The following observation can be made using Definition 1.1:

The class $\mathcal{B}_{\xi,\mu}^{\nu,\rho}(\ell, \mathcal{L}, \eta)$ is a generalization of previously proposed classes. Giving explicit attributes to $\mathcal{L}$ and ν for this class, we obtain the following subclasses:

I. In the class $\mathcal{B}_{\xi,\mu}^{\nu,\rho}(\ell, \mathcal{L}, \eta)$, if $\mathcal{L} = 0$, $\nu = 0$ and $\Im = 0$, then we obtain

$-\mathcal{R}e\left\{\frac{\zeta(g\acute{(}\zeta))}{g(\zeta)}\right\} > \eta,$

it was first introduced in [16] and reduced to the class $T_{s^*(\eta)}$.

II. In the class $B^{\nu,\rho}_{\xi,\mu}(\ell,\mathcal{L},\eta)$, if $\mathcal{L}=0, \nu=0$ and $\Im=0$ then we obtain

$-\mathcal{R}e\left\{\frac{\zeta\left((g(\zeta))\right)'}{f(\zeta)}\right\} > \eta,$

it was first introduced in [4] and reduced to the class $T_{s(\eta)}$.

2. Preliminaires

The following lemmas are necessary for you to discuss the main results.

Lemma 2.1. [9] Let $\mathcal{M}$ be an analytic function in unit disk ψ with $\mathcal{M}(0) = 0$. If $|\mathcal{M}(z)|$ attains its maximum quantity on $|\zeta| = \upsilon < 1$ at $\zeta_0 \in \psi$, then $\zeta_0\mathcal{M}'(\zeta_0) = k\mathcal{M}(\zeta_0)$, where $k \in \mathbb{R}$ and $k \geq 1$.

Lemma 2.2. [14] Let $\mathrm{P} \subset \mathbb{C}$ and assume that $\varphi(\zeta): \mathbb{C}^2 \times \psi \longrightarrow \mathbb{C}$ that satisfies $\Phi(ix, y; \zeta) \notin \mathrm{P}$ for all $\zeta \in \psi$, and for all $x, y \in \mathbb{R}$ such that $y \leq -(1+x^2)/2$. It $q(z) = 1 + q_1\zeta + q_2\zeta^2 + \cdots$ is an analytic function and $\varphi(q(\zeta), \zeta q'(\zeta); \zeta) \in \mathrm{P}$ for all $\zeta \in \psi$, then $\mathcal{R}e\{q(\zeta)\} > 0$.

Lemma 2.3 [7] Let Ω be an analytic function with $\Omega(0) = 1$. Suppose that there exists $\zeta_0 \in \psi$ such that $\mathcal{R}e\{\Omega(\zeta)\} > 0$ $(\zeta| < |\zeta_0|)$, $\mathcal{R}e\{\Omega(\zeta_0)\} = 0$ and $\Omega(\zeta) \neq 0$. Then we have $\Omega(z) = ic (c \neq 0)$ and $\frac{\zeta_0\Omega'(\zeta_0)}{\Omega(\zeta_0)} = i\frac{\kappa}{2}\left(c + \frac{1}{c}\right)$, where κ is a real number with $\kappa \geq 1$.

3. Main Results

Theorem 3.1. Let $0 \leq \varphi < 1$ and $0 \leq \Im < 1$. If $(g(\zeta))^\tau \in \mathrm{T}$ satisfies the following inequality

$$\mathcal{R}e\left\{\frac{\zeta\left(\mathcal{H}^{\nu,\rho}_{\xi,\mu}g(z)^\tau\right)'}{\left(\mathcal{H}^{\nu,\rho}_{\xi,\mu}g(z)^\tau\right)^{1-\Im}\left(\mathcal{H}^{\nu,\rho}_{\xi,\mu}g(z)^\tau\right)^{\Im}}\left[1 + \frac{\zeta\left(\mathcal{H}^{\nu,\rho}_{\xi,\mu}g(z)^\tau\right)''}{\left(\mathcal{H}^{\nu,\rho}_{\xi,\mu}g(z)^\tau\right)'} - (1-\Im)\zeta\frac{\zeta\left(\mathcal{H}^{\nu,\rho}_{\xi,\mu}g(z)^\tau\right)'}{\left(\mathcal{H}^{\nu,\rho}_{\xi,\mu}g(z)^\tau\right)} - \Im\zeta\frac{\zeta\left(\mathcal{H}^{\nu,\rho}_{\xi,\mu}f(z)^\tau\right)'}{\left(\mathcal{H}^{\nu,\rho}_{\xi,\mu}f(z)^\tau\right)}\right]\right\} > \varphi\left(\varphi + \frac{1}{2}\right) + \left(\varphi(\Im - 1) - \frac{1}{2}\right).$$

Then $\left(g(\zeta)\right)^\tau \in \mathcal{B}^{\nu,\rho}_{\xi,\mu}(\ell,\mathcal{L},\varphi)$.

Proof. The definition of the function $\mathrm{E}(\zeta)$ is

$$\frac{\zeta\left(\mathcal{H}^{\nu,\rho}_{\xi,\mu}g(z)^\tau\right)'}{\left(\mathcal{H}^{\nu,\rho}_{\xi,\mu}g(z)^\tau\right)^{1-\Im}\left(\mathcal{H}^{\nu,\rho}_{\xi,\mu}f(z)^\tau\right)^{\Im}} = -\varphi + (\varphi - 1)\mathrm{E}(\zeta), \quad (5)$$

such that the analytic function $\mathrm{E}(z) = 1 + \mathrm{E}_1\zeta + \mathrm{E}_2\zeta^2 + \ldots$

Now, when we differentiate (5) logarithmically with regard to ζ, we get

$$\varphi + (\varphi - 1)\mathrm{E}(\zeta)\left(1 - \frac{\zeta\left(\mathcal{H}^{\nu,\rho}_{\xi,\mu}g(z)^\tau\right)''}{\left(\mathcal{H}^{\nu,\rho}_{\xi,\mu}g(z)^\tau\right)'} + (1-\Im)\frac{\zeta\left(\mathcal{H}^{\nu,\rho}_{\xi,\mu}g(z)^\tau\right)'}{\left(\mathcal{H}^{\nu,\rho}_{\xi,\mu}g(z)^\tau\right)} + \Im\frac{\zeta\left(\mathcal{H}^{\nu,\rho}_{\xi,\mu}f(z)^\tau\right)'}{\left(\mathcal{H}^{\nu,\rho}_{\xi,\mu}f(z)^\tau\right)}\right)$$
$$= (\Im - 1)\left(\varphi + (1-\varphi)\mathrm{E}(\zeta)\right) + (1-\varphi)\zeta\mathrm{E}'(\zeta). \quad (6)$$

From (5) and (6), we obtain

$$\left(\frac{\zeta\left(\mathcal{H}^{\nu,\rho}_{\xi,\mu}g(z)^\tau\right)'}{\left(\mathcal{H}^{\nu,\rho}_{\xi,\mu}g(z)^\tau\right)^{1-\Im}\left(\mathcal{H}^{\nu,\rho}_{\xi,\mu}g(z)^\tau\right)^{\Im}}\right)\left(1 - \frac{\zeta\left(\mathcal{H}^{\nu,\rho}_{\xi,\mu}g(z)^\tau\right)''}{\left(\mathcal{H}^{\nu,\rho}_{\xi,\mu}g(z)^\tau\right)'} + (1-\Im)\frac{\zeta\left(\mathcal{H}^{\nu,\rho}_{\xi,\mu}g(z)^\tau\right)'}{\left(\mathcal{H}^{\nu,\rho}_{\xi,\mu}g(z)^\tau\right)} + \Im\frac{\zeta\left(\mathcal{H}^{\nu,\rho}_{\xi,\mu}f(z)^\tau\right)'}{\left(\mathcal{H}^{\nu,\rho}_{\xi,\mu}f(z)^\tau\right)}\right) = (\Im - 1)(\varphi + (1-\varphi)\mathrm{E}(\zeta) + (1-\varphi)\zeta\,\mathrm{E}'(\zeta). \quad (7)$$

Using the same method as in (7), we obtain

$$\left(\frac{\zeta\left(\mathcal{H}^{\nu,\rho}_{\xi,\mu}g(z)^\tau\right)'}{\left(\mathcal{H}^{\nu,\rho}_{\xi,\mu}g(z)^\tau\right)^{1-\Im}\left(\mathcal{H}^{\nu,\rho}_{\xi,\mu}f(z)^\tau\right)^{\Im}}\right)^2\left(-\frac{\zeta\left(\mathcal{H}^{\nu,\rho}_{\xi,\mu}g(z)^\tau\right)'}{\left(\mathcal{H}^{\nu,\rho}_{\xi,\mu}g(z)^\tau\right)^{1-\Im}\left(\mathcal{H}^{\nu,\rho}_{\xi,\mu}g(z)^\tau\right)^{\Im}}\right)$$
$$\left(1 - \frac{\zeta\left(\mathcal{H}^{\nu,\rho}_{\xi,\mu}g(z)^\tau\right)''}{\left(\mathcal{H}^{\nu,\rho}_{\xi,\mu}g(z)^\tau\right)'} + (1-\Im)\frac{\zeta\left(\mathcal{H}^{\nu,\rho}_{\xi,\mu}g(z)^\tau\right)'}{\left(\mathcal{H}^{\nu,\rho}_{\xi,\mu}g(z)^\tau\right)} + \Im\frac{\zeta\left(\mathcal{H}^{\nu,\rho}_{\xi,\mu}f(z)^\tau\right)'}{\left(\mathcal{H}^{\nu,\rho}_{\xi,\mu}f(z)^\tau\right)}\right)$$
$$= (1-\Im)\zeta\mathrm{E}'(\zeta) + (1-\varphi)^2\mathrm{E}^2(\zeta) + (1-\varphi)[2\varphi + (\Im - 1)]\mathrm{E}(\zeta) + \varphi^2 + \varphi(\Im - 1). \quad (8)$$
$$= \mathcal{F}\left(\mathrm{E}(\zeta), \zeta\mathrm{E}'(\zeta); \zeta\right),$$

where

$$\mathcal{F}(m, n; \zeta) = (1-\Im)n + (1-\varphi)^2m^2 + (1-\varphi)[2\varphi + (\Im - 1)]m + \varphi^2 + \varphi(\Im - 1).$$
$$\mathcal{R}(\mathcal{F}(i\chi, y; \zeta) = (1-\Im)y - (1-\varphi)^2\chi^2 + \varphi^2 + \varphi(\Im - 1) + \varphi(\Im - 1)$$
$$\leq -\frac{1}{2}(1-\Im)(1+\chi^2) - (1-\varphi)^2\chi^2 + \varphi^2 + \varphi(\Im - 1) + \varphi(\Im - 1)$$
$$= -\frac{1}{2}(1-\varphi) - (1-\varphi)(\frac{1}{2}(1-\varphi)\chi^2 + \varphi(\Im - 1) + \varphi^2.$$

Assume that $\mathrm{K} = \{\kappa: \mathcal{R}(\omega) > \varphi\left(\varphi + \frac{1}{2}\right) + (\varphi(\Im - 1) - \frac{1}{2}\}$.

Then $\mathcal{F}\left(E(\zeta), \zeta E'(\zeta); \zeta\right) \in \mathrm{K}$ and $F(i\chi, y; \zeta) \notin \mathrm{K}$.

By applying Lemma 2.2, we have $\mathcal{Re}(\mathrm{E}(\zeta)) > 0$, that is $\left(g(\zeta)\right)^{\tau} \in \mathcal{B}_{\xi,\mu}^{\nu,\rho}(\ell,\mathfrak{I},\varphi)$.
The proof is complete.
With $\mathfrak{I} = 0, \nu = 0,$ and $\varphi = 0$, we apply Theorem 3.1 to get the following outcome:
Corollary 3.2. If $(g(\zeta))^{\tau} \in \mathrm{T}$ satisfies the following inequality
$\mathcal{Re}\{\frac{\zeta\,(g(z)^{\tau})'}{(g(z)^{\tau})}[1 + \frac{\zeta\,(g(z)^{\tau})''}{(g(z)^{\tau})'} - \frac{\zeta^2\,(g(z)^{\tau})'}{(g(z)^{\tau})}]\} > -\frac{1}{2}$,
then $\left(g(\zeta)\right)^{\tau} \in \mathcal{B}_{\xi,\mu}^{0,\rho}(\ell,0,0)$.
For $\mathfrak{I} = 1, \varphi = 1,$ and $\nu = 0$, Theorem 3.1 gives.
Corollary 3.3. If $(g(\zeta))^{\tau} \in \mathrm{T}$ satisfies the following inequality
$\mathcal{Re}\{\frac{\zeta\,(g(z)^{\tau})'}{(g(z)^{\tau})}[1 + \frac{\zeta\,(g(z)^{\tau})''}{(g(z)^{\tau})'} - \zeta^2\,\frac{(g(z)^{\tau})'}{(g(z)^{\tau})}]\} > 1$,
then $g(\zeta)^{\tau} \in \mathcal{B}_{\xi,\mu}^{0,\rho}(\ell,1,1)$.
Theorem 3.4. If $(g(\zeta))^{\tau} \in \mathrm{T}$ and satisfies the following inequality.

$$\mathcal{Re}\{(1-\mathfrak{I})\frac{\zeta\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}g(\zeta)^{\tau}\right)'}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}g(\zeta)^{\tau}\right)} - \mathfrak{I}\frac{\zeta\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}f(\zeta)^{\tau}\right)'}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}f(\zeta)^{\tau}\right)} + \frac{\zeta\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}g(\zeta)^{\tau}\right)''}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}g(\zeta)^{\tau}\right)'}\} < 2(1-\mathfrak{I}) - \varphi,$$

then

$$-\mathcal{Re}\left\{\frac{\zeta\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}g(z)^{\tau}\right)'}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}g(z)^{\tau}\right)^{1-\mathfrak{I}}\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}f(z)^{\tau}\right)^{\mathfrak{I}}}\right\} > \eta$$
$$= \frac{1}{1+(1-\mathfrak{I})-2\varphi}, (\zeta \in \psi).$$

where $0 \le \mathfrak{I} < 1$ and $\frac{2(1-\mathfrak{I})-1}{2} \le \varphi < 1 - \mathfrak{I}$.
Proof. The definition of the function $\Omega(z)$ in ψ is as follows:

$$-\frac{\zeta\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}g(z)^{\tau}\right)'}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}g(z)^{\tau}\right)^{1-\mathfrak{I}}\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}f(z)^{\tau}\right)^{\mathfrak{I}}} = \eta + (1-\eta)\Omega(\zeta). \quad (9)$$

With $\eta = \frac{1}{1+2(1-\mathfrak{I})-2\varphi}$.
Given that $\Omega(0) = 1$, it follows that $\Omega(\zeta)$ is analytic in ψ.

$$(1-\mathfrak{I})\frac{\zeta\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}g(\zeta)^{\tau}\right)'}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}g(\zeta)^{\tau}\right)} - \mathfrak{I}\frac{\zeta\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}f(\zeta)^{\tau}\right)'}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}f(\zeta)^{\tau}\right)} + \frac{\zeta\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}g(\zeta)^{\tau}\right)''}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}g(\zeta)^{\tau}\right)'}$$
$$= (1-\mathfrak{I}) - \mathfrak{I}\frac{\zeta\Omega'(\zeta)(\eta-1)}{\eta+(1-\eta)\Omega(\zeta)} \quad (10)$$

If $\zeta_0 \in \psi$, then $\mathcal{Re}\{\Omega(\zeta)\} > 0$, $(|\zeta| < |\zeta_0|)$, $\mathcal{Re}\{\Omega(\zeta_0)\} = 0, \Omega(\zeta) \neq 0$.
Thus, using Lemma 2.3, we have $\Omega(\zeta) = ic\ (c \neq 0)$, and
$\frac{\zeta_0\Omega'(\zeta_0)}{\Omega(\zeta_0)} = i\frac{\kappa}{2}\left(c + \frac{1}{c}\right). (\kappa \ge 1)$
Thus, we might deduce that

$$(1-\mathfrak{I})\frac{\zeta_0\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}g(\zeta_0)^{\tau}\right)'}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}g(\zeta_0)^{\tau}\right)} - \mathfrak{I}\frac{\zeta_0\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}f(\zeta_0)^{\tau}\right)'}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}f(\zeta_0)^{\tau}\right)} + \frac{\zeta_0\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}g(\zeta_0)^{\tau}\right)''}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}g(\zeta_0)^{\tau}\right)'}$$
$$= (1-\mathfrak{I}) - \mathfrak{I}\frac{\zeta_0\Omega'(\zeta_0)(\eta-1)}{\eta+(1-\eta)\Omega(\zeta_0)} = (1-\mathfrak{I}) - \mathfrak{I}\frac{\kappa(1-\mathfrak{I})(1+c^2)}{2(\mathfrak{I}+i(1-\mathfrak{I})c)}.$$

Additionally, we obtain

$$\mathcal{Re}\{(1-\mathfrak{I})\frac{\zeta_0\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}g(\zeta_0)^{\tau}\right)'}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}g(\zeta_0)^{\tau}\right)} - \mathfrak{I}\frac{\zeta_0\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}f(\zeta_0)^{\tau}\right)'}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}f(\zeta_0)^{\tau}\right)} + \frac{\zeta_0\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}g(\zeta_0)^{\tau}\right)''}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}g(\zeta_0)^{\tau}\right)'}\}$$
$$= (1-\mathfrak{I}) - \mathfrak{I}\frac{\kappa(1-\mathfrak{I})(1+c^2)}{2(\mathfrak{I}+i(1-\mathfrak{I})c)} \ge (1-\mathfrak{I}) - \frac{\kappa(1-\mathfrak{I})}{2} \quad (12)$$
$$\ge 2(1-\mathfrak{I}) - \varphi.$$

This contradicts what we thought. Hence, for all $\zeta \in \psi$, $Re\{\Omega(\zeta)\} > 0$. So

$$-\mathcal{Re}\left\{\frac{\zeta\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}g(z)^{\tau}\right)'}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}g(z)^{\tau}\right)^{1-\mathfrak{I}}\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}f(z)^{\tau}\right)^{\mathfrak{I}}}\right\} > \eta$$
$$= \frac{1}{1+(1-\mathfrak{I})-2\varphi}, (\zeta \in \psi).$$

By applying Theorem 3.4 $\varphi = \frac{(2(1-\mathfrak{I})-1)}{2}$, we may get

Corollary 3.5. If $(g(\zeta))^{\tau} \in \mathrm{T}$ and satisfies the following inequality

$$\mathcal{Re}\{(1-\mathfrak{I})\frac{\zeta\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}g(\zeta)^{\tau}\right)'}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}g(\zeta)^{\tau}\right)} - \mathfrak{I}\frac{\zeta\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}f(\zeta)^{\tau}\right)'}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}f(\zeta)^{\tau}\right)} + \frac{\zeta\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}g(\zeta)^{\tau}\right)''}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}g(\zeta)^{\tau}\right)'}\} < \frac{2(1-\mathfrak{I})+1}{2}$$

then

$$-\mathcal{R}e\left\{\frac{\zeta\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(z)^{\tau}\right)'}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(z)^{\tau}\right)^{1-\mathfrak{I}}\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}f(z)^{\tau}\right)^{\mathfrak{I}}}\right\} > \frac{1}{1+\mathfrak{I}},$$

where $0 \leq \mathfrak{I} < 1$.

For and in Theorem 3.5, we get:

If $\mathfrak{I} = 0$ and $\nu = 0$ in Theorem 3.4, we obtain

Corollary 3.6. If $(\mathcal{g}(\zeta))^{\tau} \in \mathrm{T}$ and satisfies the following inequality.

$$\mathcal{R}e\left\{\frac{\zeta\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(\zeta)^{\tau}\right)'}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(\zeta)^{\tau}\right)} + \frac{\zeta\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(\zeta)^{\tau}\right)''}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(\zeta)^{\tau}\right)'}\right\} < 2 - \varphi.$$

then

$$-\mathcal{R}e\left\{\frac{\zeta\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(z)^{\tau}\right)'}{\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(z)^{\tau}}\right\} > \frac{1}{2-2\varphi},$$

where $\frac{1}{2} \leq \varphi < 1$.

As a result, this corollary reduces to the outcome displayed in ([4], Corollary 2.4).

Theorem 3.7. Let $0 \leq \mathrm{A} < 1$ and $0 \leq \mathfrak{I} < 1$. If $(\mathcal{g}(\zeta))^{\tau} \in \mathrm{T}$ satisfies

$$\left|1 + \zeta\frac{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(z)^{\tau}\right)''}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(z)^{\tau}\right)'} - (1-\mathfrak{I})\frac{\zeta\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(z)^{\tau}\right)'}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(z)^{\tau}\right)} - \mathfrak{I}\frac{\zeta\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}f(z)^{\tau}\right)'}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}f(z)^{\tau}\right)} - \eta\left(\frac{\zeta\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(\zeta)^{\tau}\right)'}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(\zeta)^{\tau}\right)^{1-\mathfrak{I}}\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}f(\zeta)^{\tau}\right)^{\mathfrak{I}}}\right)\right| < \eta(1-A) + \frac{(1-A)}{(2-A)}. \quad (13)$$

Then $\left(\mathcal{g}(\zeta)\right)^{\tau} \in \mathcal{B}_{\xi,\mu}^{\nu,\rho}(\ell,\mathcal{L},\varphi)$.

Proof. Define $\mathcal{M}(z)$ by

$$\frac{\zeta\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(\zeta)^{\tau}\right)'}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(\zeta)^{\tau}\right)^{1-\mathfrak{I}}\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}f(\zeta)^{\tau}\right)^{\mathfrak{I}}} = -1 + (\mathrm{A}-1)\mathcal{M}(\zeta), \quad (14)$$

consequently, $\mathcal{M}(0) = 0$ and $\mathcal{M}(\mathrm{z})$ is an analytic function when we logarithmically differentiate (14) in relation to ζ, we obtain

$$1 + \zeta\frac{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(z)^{\tau}\right)''}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(z)^{\tau}\right)'} - (1-\mathfrak{I})\frac{\zeta\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(z)^{\tau}\right)'}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(z)^{\tau}\right)} - \mathfrak{I}\frac{\zeta\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}f(z)^{\tau}\right)'}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}f(z)^{\tau}\right)} = \frac{(1-\mathrm{A})\zeta\mathcal{M}'(\zeta)}{1+(1-\mathrm{A})\mathcal{M}(\zeta)}. \quad (15)$$

When we use (13) in (14), we obtain

$$1 + \zeta\frac{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(z)^{\tau}\right)''}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(z)^{\tau}\right)'} - (1-\mathfrak{I})\frac{\zeta\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(z)^{\tau}\right)'}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(z)^{\tau}\right)} - \mathfrak{I}\frac{\zeta\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}f(z)^{\tau}\right)'}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}f(z)^{\tau}\right)} - \eta\left(\frac{\zeta\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(\zeta)^{\tau}\right)'}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(\zeta)^{\tau}\right)^{1-\mathfrak{I}}\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}f(\zeta)^{\tau}\right)^{\mathfrak{I}}}\right) = \eta\,(1-\mathrm{A})\Psi(\zeta) + \frac{(1-\mathrm{A})\zeta\mathcal{M}'(\zeta)}{1+(1-\mathrm{A})\mathcal{M}(\zeta)}. \quad (16)$$

Let $\zeta_0 \in \psi$ such that

$\max_{|\zeta|<|\zeta_0|}|\mathcal{M}(z)| = |\mathcal{M}(\zeta_0)|$,

and Lemma 2.1 application, we obtain

$\zeta_0\mathcal{M}(\zeta_0) = \kappa\mathcal{M}(\zeta_0), (\kappa \geq 1)$

Putting $\zeta = \zeta_0$ in equation (16) and setting $\mathcal{M}(\zeta) = e^{i\theta}$ $(0 \leq \theta < 2\pi)$, we obtain

$$\left|1 + \zeta\frac{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(z)^{\tau}\right)''}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(z)^{\tau}\right)'} - (1-\mathfrak{I})\frac{\zeta\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(z)^{\tau}\right)'}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(z)^{\tau}\right)} - \mathfrak{I}\frac{\zeta\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}f(z)^{\tau}\right)'}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}f(z)^{\tau}\right)} - \eta\left(\frac{\zeta\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(\zeta)^{\tau}\right)'}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(\zeta)^{\tau}\right)^{1-\mathfrak{I}}\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}f(\zeta)^{\tau}\right)^{\mathfrak{I}}}\right)\right| = \left|\eta(1-\mathrm{A})e^{i\theta} + \frac{(1-\mathrm{A})\kappa e^{i\theta}}{1+(1-\mathrm{A})e^{i\theta}}\right| \quad (17)$$

$\geq \mathcal{R}e(\eta(1-\mathrm{A}) + \frac{(1-\mathrm{A})\kappa}{1+(1-\mathrm{A})e^{i\theta}}$

$> \eta(1-\mathrm{A}) + \frac{(1-\mathrm{A})}{(2-\mathrm{A})}$.

This runs counter to our assumption (13). Thus, in ψ, we have $|\mathcal{M}(\zeta)| < 1$. Lastly, we have

$$\left|\frac{\zeta\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(\zeta)^{\tau}\right)'}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(\zeta)^{\tau}\right)^{1-\mathfrak{I}}\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}f(\zeta)^{\tau}\right)^{\mathfrak{I}}} + 1\right| = |(1-\mathrm{A})\mathcal{M}(\zeta)| = (1-\mathrm{A})|\mathcal{M}(\zeta)| < 1 - \mathrm{A}. \quad (18)$$

Notice that the condition (18) is equivalent to

$$-\mathcal{R}e\left\{\frac{\zeta\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(\zeta)^{\tau}\right)'}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(\zeta)^{\tau}\right)^{1-\mathfrak{I}}\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}f(\zeta)^{\tau}\right)^{\mathfrak{I}}}\right\} > \eta,$$

$\left(\mathcal{g}(\zeta)\right)^{\tau} \in \mathcal{B}_{\xi,\mu}^{\nu,\rho}(\ell,\mathcal{L},\eta)$.

According to Theorem 3.7, we have for $\mathfrak{I} = 0, \nu = 0, \mathrm{A} = 0$ and $\eta = 0$.

Corollary 3.8. If $(\mathcal{g}(\zeta))^{\tau} \in \mathrm{T}$ satisfies the following inequality

$$\left|1 + \zeta\frac{\zeta\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(\zeta)^{\tau}\right)''}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(\zeta)^{\tau}\right)'} - \frac{\zeta\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(\zeta)^{\tau}\right)'}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(\zeta)^{\tau}\right)}\right| < \frac{1}{2},$$

then $(\mathcal{g}(\zeta))^{\tau} \in \mathrm{T}$.

We obtain for $\mathfrak{I} = 1, \upsilon = 0,$ and $\eta = 1$ in Theorem 3.7.

As a result, this Corollary reduces to the outcome displayed in [[6], Corollary 7].

Corollary 3.9. If $(\mathcal{g}(\zeta))^\tau \in \mathrm{T}$ satisfies the following inequality

$$\left|1+\zeta\frac{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(z)^\tau\right)''}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(z)^\tau\right)'}-\frac{\zeta\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(z)^\tau\right)'}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(z)^\tau\right)}-\frac{\zeta\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}f(z)^\tau\right)'}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}f(z)^\tau\right)}\right|<\frac{(1-A)(3-\mathrm{A})}{(2-A)}.$$

then $(\mathcal{g}(\zeta))^\tau \in \mathcal{B}_{\xi,\mu}^{0,\rho}(\ell,1,\mathrm{A})$.

Additionally, when we enter $\mathrm{A}=0$ in Corollary 3.9, we obtain

Corollary 3.10. If $(\mathcal{g}(\zeta))^\tau \in \mathrm{T}$ satisfies the following inequality

$$\left|1+\zeta\frac{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(z)^\tau\right)''}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(z)^\tau\right)'}-\frac{\zeta\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(z)^\tau\right)'}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}\mathcal{g}(z)^\tau\right)}-\frac{\zeta\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}f(z)^\tau\right)'}{\left(\mathcal{H}_{\xi,\mu}^{\nu,\rho}f(z)^\tau\right)}\right|<\frac{3}{2},$$

then $\left(\mathcal{g}(\zeta)\right)^\tau \in \mathcal{B}_{\xi,\mu}^{0,\rho}(\ell,1,0)$.

4. Conclusions

This study investigates a Bazilevic function associated with the class of meromorphic functions in a unit disk, which is defined by a differential operator. We obtained numerous crucial discoveries by exploring the geometric and analytic features of these functions, which help to our understanding of their structure. The findings show how Bazilevic-type conditions affect the mapping properties and growth of meromorphic functions. These findings not only expand current theorems, but also suggest new directions for future research in geometric function theory, particularly in the study of differential subordinations and integral operators within meromorphic function classes

References

[1] Abdulnabi, F.F., Al-Janaby, H.F., Ghanim, F., Alb Lupas, A., Some Results on Third-Order Differential Subordination and Differential Superordination for Analytic Functions Using a Fractional Differential Operator, Mathematics 11,2023, 4021.

[2] Abdul Ameer M., A. R. Juma and R. Al-Saphory, On differential subordination of higher-order derivatives of multivalent functions, Journal of Physics: Conference Series, 1818 (012188), 2021, 1-10.

[3] Attiya, A.A., Aouf M.K., Ali E.E., Yassen M.F., Differential subordinationand superordination results associated with Mittag Leffler function, Mathematics, 9, 2021,226.

[4] Cho N. E. and Owa S. , Sufficient Condiions for meromorphic starlikeness and close-to-convexity of order α, IJMMS, 26(5), 2001,317-319.

[5] Duren P. L., Univalent Functions, Grundlehren der Mathematishen Wissenschaften 259, Springer-Verlag, 1983.

[6] Goyal P. S and Prajapat J. K., A New class of meromorphic multivalent functions involving certain linear operator, Tamsui Oxford Journal of Math sciences , 25(2), 2009,167-176.

[7] Grunsky H., Koeffizientenbedingungen fur schlicht abbildende meromorphe Funktionen, Math. Zeit., 45,1939, 29-61.

[8] Juma, A.R.S., Saloomi, M.H., Coefficient Estimates for Subclasses of Regular Functions, Iraqi Journal of Science,59(3C), 2018, 1710-1716.

[9] Jack I. S., Functions starlike and convex of order δ, J. Londan Math. Soc., 3(2), 1971, 469-474.

[10] Juma A.R., M.S. Abdulhussain, S.N. Al-khafaji, Certain subclass of p-valent meromorphic Bazilevic functions defined by fractional q-calculus operators, J. Nonlinear Anal.,(2), 2018, 223-230.

[11] Juma A.R., M. Abdulhussain, and S. N. Al-khafaji, Faber polynomial coefficient estimates for subclass of analytic Bi Bazilevic functions defined by dfferential operator, Baghdad Science Journal, 16 (1), 2019, 248-252.

[12] Koebe P., Ueber die Uniformisierung beliebiger analytischer Kurven, Nachrichten von der Gesellschaft der Wiss. zu Göttingen, Mathematisch-Phys. Klasse,1907, 191-210.

[13] I.E. Bazilevic, On a case of integrability in quadratures of the Loewner Kufarev equation, Mat. Sb. (N.S.) 37(79),1955,471–476.

[14[Miller S. S. and Mocanu P. T., Differential subordinations and inequalities in the complex plane, J. Differ. Equations. , 67, 1987, 199-211.

[15] Miller S., Mocanu, P.T., Subordinants of differential superordinations Complex Var. 2003, 48, 815–826.

[16] Miller S., Mocanu, P.T., Differential subordinations and univalent functions, Michigan Math. J., 28(2), 1981,157-172.

[17] Oros, G.I. Strong differential superordination. Acta Univ. Apulensis. Math.-Inform., 19,2009, 101–106.

[18] Shehab N. H., Juma, A.R.S., Third Order Differential Subordination for Analytic Functions Involving Convolution Operator, Baghdad Science Journal, 2021, 581-592.

[19] Shehab, N.H., Juma, A.R.S., Application of Quasi Subordination Associated with Generalized Sakaguchi Type Functions, Iraqi Journal of Science, 62(12), 2021, 4885-4891.

[20] Zayed, H., Bulboacă, T., Sandwich results for higher order fractional derivative operators, Mat. Stud., 49,2018, 52–66.